Signal-setting and ATIS in traffic networks

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ABSTRACT

This paper aims at analysing the role that can be played in traffic networks by Advanced Traveller Information Systems (ATIS) in conjunction with signal setting (SS) design. SS has been widely studied as an optimization problem but not all optimal settings are feasible, because of equilibrium and stability constraints to be respected. In this paper the role of ATIS is assessed with reference to its suitability in inducing SS optima and/or in stabilizing equilibrated solutions.

Keywords: *ATIS, Signal Setting, ATMS; dynamics; Stability; Optimum; Equilibrium*

INTRODUCTION

This paper aims at analysing the role that can be played in traffic networks by Advanced Traveller Information Systems (ATIS) in conjunction with signal setting (SS) design. This vast field has received relatively scarce attention in the literature and has been mainly viewed as an application area for (within-day) dynamic control problems [1]. The approach followed in this paper is to frame both SS and ATIS in a coherent theoretical model and to explore some of the potential of this frame. In particular, this paper explores the role of SS and ATIS with respect to equilibrium and stability. The analyses are supported by numerical examples.

It is well known that optimisation of signal parameters represents one of the available tools toward the improvement of traffic network performances. Traffic signals can be optimised with respect to local criteria (like as equisaturation, [2]) or with respect to network criteria (like as minimisation of total travellers' delay). It is also well known that, in congested networks, the SS problem is meaningless if not consistent with equilibrium flows [3] [4] [5] [6] [7] [8] [9]. Thus, sub-optimal (say, *second-best*) patterns, consistent with equilibrium constraints, have to be accepted for SS. In this context, one of the recurrent temptations is to use information provided by ATIS in order to move traffic systems toward some desired pattern ([10] [11] [12] [13] [14]), as for instance the first-best solution obtained by the sole SS problem. This paper will show that this is not a suitable aim for general cases and poor results can be obtained.

Moreover, equilibrium is not the only concern in SS. Because of stability issues, solutions could be unrealistic also if consistent with equilibrium, as discussed for instance by [15] with reference to some usual policies and as more recently highlighted by [16]. This means that a further constraint has to be imposed, aimed at ensuring not only equilibrium but also stability.

This leads to a sort of *third-best* (but fully feasible) solution. This paper also discusses if ATIS could help to cope with stability, in order to allow for more effective solutions.

With these aims, this work develops a modelling framework able to deal with the SS problem by considering both the constraints imposed by the equilibrium assignment and the stability of this equilibrium under ATIS.

SIGNAL SETTING

SS is recognised as a suitable tool in order to optimise network performances with respect to a variety of alternative goals. Most of the existing SS approaches can be formalised as the optimisation of a proper objective function z(.):

$$\gamma^{LSS}(\boldsymbol{f}) = \text{argopt}_{\gamma}\, z(\gamma, \boldsymbol{f}) \qquad s.t.\ \gamma \in S_{\gamma}$$

where γ is the vector of signal parameters, $\boldsymbol{f}$ is the link flows vector and S_{γ} is the set of feasible (e.g. non-negative) SS.

Previous equation states a *local* signal setting (LSS) problem where the arc flows are considered as known and fixed parameters. Actually, the solution is a function of the flows, provided that different flows lead to different LSS solutions.

A more realistic system-optimum problem can be defined where the optimisation refers to both the arc flows and the signal settings. The solution of this problem can be considered as to be *global*, in the sense that both flows and signal setting are subject to optimisation and the best (γ, $\boldsymbol{f}$) couple is searched. It is also here referred to as the *unconstrained* SS problem (USS), in the sense that vectors γ and $\boldsymbol{f}$ are only subject to independent feasibility conditions and are not required to be mutually consistent. In formal terms the USS solution can be described by the equation:

$$(\gamma^{USS}, \boldsymbol{f}^{USS}) = \text{argopt}_{\gamma,f}\, z(\gamma, \boldsymbol{f})\ s.t.\ \boldsymbol{f} \in S_f,\ \gamma \in S_{\gamma}$$

where S_f is the feasibility set of link flows, mainly respecting non-negativity, demand conservation between o/d couples and flow conservation at nodes.

However, signal parameters and arc flows are not independent each other. SS can affect in a not negligible way the network performances and so the route choices. As a consequence, the consistency of SS with traffic flows has to be considered. Such an approach is sometimes called signal setting with elastic flows and more often called signal setting with assignment, provided that the assignment model is added to the optimisation process.

In absence of ATIS and for any given SS vector $\boldsymbol{\gamma}$, the assignment model can be expressed by a fixed point formulation:

$$\boldsymbol{f}_{eq} = \mathbf{f}_{NL}(\ \mathbf{c}(\boldsymbol{f}_{eq},\gamma)\ ) \qquad 1)$$

where $\mathbf{f}_{NL}(.)$ represents the network loading function mapping the relationship from costs $\boldsymbol{c}$ to link-flows $\boldsymbol{f}$. The fixed point solution can be also formalised as an implicit function:

$$\boldsymbol{f}_{Eq} = \Phi^{*}_{Eq}(\gamma) \qquad 2)$$

As a result, the SS with assignment solution (also indicated as the *Equilibrated* Signal Setting - ESS) can be described by:

$$\boldsymbol{\gamma}^{ESS} = \text{argopt}_{\gamma}\ z(\ \boldsymbol{\gamma},\ \Phi^{*}_{Eq}(\ \gamma)\ )\ s.t.\ \gamma \in S_{\gamma}$$

Actually, the ESS solution could be existent but not observable, in the sense that once reached is not kept, being unstable. In other terms, the equilibrium could be not the only constraint to be imposed.

In order to assess the stability of the equilibrium, a dynamic model representing the evolution of the system over days has to specified. According to the well known, simple but effective, exponential smoothing approach [17] [18], two dynamic equations can describe the process:

$$\boldsymbol{x}^{t} = \beta\ \mathbf{c}(\boldsymbol{f}^{t-1},\ \ \gamma) + (1-\beta)\ \boldsymbol{x}^{t-1} \qquad 3.a)$$

$$\boldsymbol{f}^{t} = \alpha\ \mathbf{f}_{NL}(\boldsymbol{x}^{t}) + (1-\alpha)\ \boldsymbol{f}^{t-1} \qquad 3.b)$$

where $\alpha \in\ ]0,1]$ and $\beta \in\ ]0,1]$ are the (time-invariant) parameters of the exponential smoothing filter and $\boldsymbol{x}^{t}$ is the vector of user-forecasted arc costs. Stability can be evaluated by means of the eigenvalues of the Jacobian matrix of the dynamic process established by equations 3.a and 3.b, $\mathbf{J}(\mathbf{x}^{t}, \mathbf{f}^{t})$, according to the approach discussed in [17]. In the specific case of SS (without ATIS), it has been shown in [16] that the stability also depends on $\boldsymbol{\gamma}$, thus, the solution of the SS problem has to be further constrained. Because of the new added constraint, the new solution (Stable Signal Setting – SSS) is in general worse with respect to the ESS solution and a fortiori worse (*third-best*) with respect to the USS one.

ROLE OF ATIS

In general cases, ATIS influence route choices and thus traffic patterns. As a consequence, ATIS influence the SS problem, at least in congested networks.

In order to analyse the impact, the models previously described in absence of ATIS have to be reformulated.

For what concerns user equilibrium, the model should take into account the role of the dispatched information ($\mathbf{g}$), here intended as the estimates of the travel times provided by a descriptive ATIS. In formal terms:

$$\boldsymbol{f}^{IS} = \lambda\ \mathbf{f}_{C}(\boldsymbol{g}) + (1-\lambda\ )\ \mathbf{f}_{U}(\ \mathbf{c}(\boldsymbol{f}^{IS},\gamma)\ ) \qquad 4)$$

where $\mathbf{f}_{C}(.)$ and $\mathbf{f}_{U}(.)$ are the network loading functions for user compliant and not compliant with the information and λ is the percentage of compliant user. The compliance λ is assumed to attain the market penetration (λ_{max}, here fixed and known) in case of fully-accurate information and to decrease according with the inaccuracy of the information, computed as the distance from the dispatched travel times and the actual ones. In formal terms:

$$\lambda = \lambda\ (\boldsymbol{g},\ \mathbf{c}(\boldsymbol{f}^{IS},\gamma),\ \lambda_{max})$$

Thus, for any given market penetration λ_{max}, the equilibrium in presence of ATIS depends on the dispatched information $\boldsymbol{g}$ and on vector $\boldsymbol{\gamma}$, as well as on the solution of a fixed-point problem. In formal terms:

$$\boldsymbol{f}^{IS} = \Phi^{*}_{IS}(\gamma,\ \boldsymbol{g}) \qquad 5)$$

where equation 5 replaces equation 2 in case of ATIS.

The previous is the revised equilibrium-constraint to be imposed in the ESS problem under ATIS. Now the problem also depends on vector $\boldsymbol{g}$ of the dispatched travel times and can be fully solved only if the joint optimisation of signal-setting and information problem is considered:

$$(\gamma_{IS}^{ESS},\ \boldsymbol{g}^{ESS}) = \text{argopt}_{\gamma,g}\ z(\ \boldsymbol{\gamma},\ \Phi^{*}_{IS}(\gamma,\ \boldsymbol{g}))$$

In principle, the value of the objective function computed for joint signal-setting-and-information $z(\gamma_{IS}^{ESS},\ \Phi^{*}_{IS}(\gamma_{IS}^{ESS},\ \boldsymbol{g}^{ESS})$ could be better than the sole signal-setting $z(\boldsymbol{\gamma},\ \Phi^{*}_{Eq}(\gamma^{ESS}))$ and typically $\gamma_{IS}^{ESS} \neq \gamma^{ESS}$. This is a relatively unexplored field of research which could contribute to the development of fully-coupled and integrated ATIS+ATMS (Advanced Traffic Management Systems) applications.

Exploration of this field is hard from a theoretical point of view; however, some special (extreme) cases are more affordable.

For instance, if the dispatched travel times are strongly inaccurate (inconsistent with the resulting equilibrium costs), they lead to a null compliance, thus $\Phi^{*}_{IS}(.) = \Phi^{*}_{Eq}(.)$ and so $\gamma_{IS}^{ESS} = \gamma^{ESS}$.

A different extreme case arises when the dispatched information is based on a fully accurate strategy. In this case the information to be dispatched no longer is a free variable (actually, $\mathbf{g}$ is obtained by link costs $\mathbf{c}$), the compliance coincides with the market penetration λ_{max} and the optimisation problem can be solved again with respect to only $\boldsymbol{\gamma}$ (given λ_{max}):

$$\gamma_{A}^{ESS} = \text{argopt}_{\gamma}\ z(\ \boldsymbol{\gamma},\ \Phi^{*}_{A\_IS}(\gamma)\ )\ s.t. \qquad \gamma \in S_{\gamma} \qquad 6)$$

where $\Phi^{*}_{A\_IS}(.)$ is the specialisation of the implicit function defined in equation 5 for the case of fully-accurate ATIS. This case can be defined as the signal setting optimisation under accurate ATIS.

Also the optimal solution γ_{A}^{ESS} is in general different from the one in absence of ATIS (γ^{ESS}) and there is no evidence that $z(\gamma_{A}^{ESS},\ \Phi^{*}_{A\_IS}(\gamma_{A}^{ESS}))$ could be better or worse than $z(\gamma^{ESS},\ \Phi^{*}_{Eq}(\gamma^{ESS}))$.

An even different question is here approached. It concerns if is it possible to appropriately design the information $\boldsymbol{g}$ in order to let the solution γ^{ESS} consistent with the user equilibrium in presence of ATIS. In other terms, the desired signal setting solution is fixed (γ^{USS} – which is the best obtainable in absence of ATIS) and the ATIS is used in order to induce a traffic pattern $\boldsymbol{f}^{IS} = \boldsymbol{f}^{USS}$, when $\mathbf{c}(\boldsymbol{f}^{USS},\gamma^{USS})$ and $\boldsymbol{f}^{USS}$ are in equilibrium.

Further than the previous equilibrating problem, the stability of the equilibrium solution under ATIS can be

considered after having extended equations 3.a and 3.b in order to consider the presence of ATIS also in the dynamic process. This allows for comparing the stability of the solutions obtained from equation 6 (γ_A^{ESS} in case of fully accurate ATIS) with the ones obtained in absence of ATIS (γ^{ESS} and γ^{SSS}).
The modelling framework previously described, together with the results about the effects of ATIS on the stability of equilibrium provided in [19] [20], are used in next section in order to assess through a numerical example the role of ATIS in SS design.

NUMERICAL EXAMPLES

Numerical examples are based on a toy network with 2 origin-destination pairs, 3 paths and 5 links. The origin/destination flows are considered to be constant over days and equal for both O/D pairs ($d_1 = d_2 = d$).

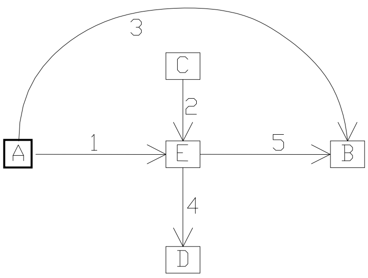

Figure 1 – The toy network

At node *E* a traffic light is placed. The signal has two stages, one associated to direction AEB and the other to direction CED. We assume as reference the green ratio (γ) associated to path AEB ($\gamma_1 = \gamma$; $\gamma_2 = 1 - \gamma$). The congestion model is taken into account by considering a binomial cost function for each link *a:*
$c_a(\boldsymbol{f}) = t^r{}_a(\boldsymbol{f}) + t^w{}_a(\boldsymbol{f})$.
The running time $t^r{}_a$ is simulated by means of a Davidson function [21], while the waiting time $t^w{}_a$ is simulated through a Webster function [22].
Different networks, characterised by different saturation levels, can be simulated by different values of the saturation flow *s* of link 1, in our case from 600 to 1000 vehic/h. For computing route choice probabilities $\boldsymbol{p}^t$ a MNL (multinomial logit) model is assumed. As objective function z(.), to be minimised in order to solve the SS problem, the total travel time on the network is used.

Table 1 – First-best and second-best signal-setting optima

Netw #	*s* vehic/h	System-optimum γ^{USS}	$z(\gamma^{USS}, \boldsymbol{f}^{USS})$	Equilibrium γ^{ESS}	$z(\gamma^{ESS}, \Phi^*_{Eq}(\gamma^{ESS}))$	ΔEq%
1	1000	0.53	157780	0.55	163480	3.6%
2	900	0.55	160230	0.57	166540	3.9%
3	800	0.56	163290	0.59	170290	4.3%
4	700	0.58	167150	0.60	174880	4.6%
5	600	0.59	172080	0.61	180470	4.9%

Table 1 shows the solutions of the optimisation problem both considering an unconstrained optimisation USS and the equilibrium-constrained solution ESS. The value ΔEq% represents the price to be paid for feasibility; it is the percentage difference between the objective function computed in ESS and USS.
As expected, the unconstrained (but unfeasible) solution is better than then equilibrium-consistent one, this phenomenon is more evident for increasing levels of saturation (from network 1 to 5).
Numerical tests have been carried out also to show the impact of trying to force toward the equilibrium by means of ATIS. The compliance elasticity model has been (conservatively) considered to be linearly decreasing from the market penetration (in case of fully-accurate information) to zero (in case of 100% inaccurate information). The market penetration has been considered very high (70%) to maximise the chances of being successful in re-equilibrating the system.
Results are reported in table 2. The values of ΔEq% are reported from table 1, the measures the difference of the ESS from the USS solution. Under the hypothesis that the ATIS is employed in order to drive the traffic patterns toward the USS solution, the table shows the value z(.) of the objective function actually reachable, the actually obtained compliance λ, the actual difference Δ_{act}% between the obtained ATIS-equilibrated solution and the USS and, finally, the advantage obtained by the ATIS-equilibrated solution with respect to the ESS (in practice, how much Δ_{act}% is smaller than ΔEq%). It is worth noting that for network 1 the equilibration is almost successful; however, the USS pattern is not reached in any case and for the most saturated network (netw 5) the information to be dispatched is so inaccurate that the compliance goes to zero and the solution actually remain the ESS one. These analyses confirm that ATIS actually is not a 100% effective tool, at least in general cases, for equilibrating system-optimum (first-best) SS solutions.

Table 2 – Equilibrating the system-optimum signal setting solution

Netw #	s vehic/h	ΔEq%	ATIS with ***g*** toward USS z(.)	λ	Δ_{act}%	Advantage
1	1000	3.6%	158043	34%	0.2%	95%
2	900	3.9%	160986	33%	0.5%	88%
3	800	4.3%	165555	32%	1.4%	68%
4	700	4.6%	174010	31%	4.1%	11%
5	600	4.9%	180470	0%	4.9%	0%

As already discussed, the equilibrium-consistent solution could be not kept by the system unless it is also a (locally) stable attractor for the dynamic process. Otherwise the stable solution (γ^{SSS}) can be significantly different from γ^{ESS}. Stability depends on parameters α and β of the dynamic process, fixed in the following both at 0.55. In case of ATIS, it depends also on λ_{max}.
The following table 3 presents the difference in terms of objective function between the ESS and the SSS solutions. This difference also is the potential advantage obtainable by the stabilisation of the equilibrated-only solution; it assumes significant values in table 3 also in cases of mild and medium-saturated conditions (up to 40%). For the most saturated network the potential advantage is infinitely great because in absence of ATIS the stable solution can't be reached. Table 3 also reports

the minimum required market penetration λ_{max} allowing for the stabilisation of the ESS solution.

Table 3 – Stabilisation of the equilibrated-only solution

Netw #	s vehic/h	z(.) ESS	z(.) SSS	Diff%	Min. market penetr. λ_{max}
1	1000	163480	164870	1%	6.7%
2	900	166540	169140	2%	7.5%
3	800	170290	185040	9%	8.1%
4	700	174880	244150	40%	8.3%
5	600	180470	N.A.	∞	14.0%

The goal of stabilising the system seems to be easily attained, also in correspondence of small values of the market penetration and for high saturation levels.

CONCLUSIONS

Signal setting problems have been widely studied in the literature. Some characteristics of the problem are widely recognised, while other are less studied, like the need of incorporating equilibrium stability constraints in order to actually ensure feasibility.

In this paper the problem is formalised also considering (day-to-day) dynamics and ATIS. Some new modelling issues have been stated and resolved by the authors, while some others, obtained by the authors in previous researches, have been here recalled, adapted and harmonised in a consistent theoretical framework. The work carried out so far has allowed to obtain several interesting results, clarifying several points through a rigorous formalisation of ATIS-related topics. In particular:

1) the problem of a both *equilibrium-consistent* and *stable* solutions exists and can lead to second and third-best solutions with respect to the sole system-optimum SS problem; moreover, feasible solutions can be significantly poorer (e.g. in terms of total travel time on the network) than unconstrained ones;
2) ATIS are only partially suitable for recovering the gap between the second-best (equilibrated) and the first-best (unconstrained) SS solutions;
3) contrary to previous point 2, ATIS are powerful tools for the stabilisation of (unstable) equilibrated solutions; they allows for recovering the gap between third-best (stable equilibrium) and second-best (equilibrium) SS solutions.

ACKNOWLEDGEMENTS

This work has been partially supported under PRIN national grant n. 2007R9CSXY_004 (2007) and UNISA local grant n. ORSA091208 (2009).

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